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Prediction of Pressurant Mass Requirements for Axisymmetric Liquid Hydrogen Tanks

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Experimental data from several test series are compared to an existing correlation that predicts the amount of pressurant gas mass required to expel liquid hydrogen from axisymmetric tanks. It was necessary to use an alternative definition of the tank equivalent diameter to accommodate thermal mass in the tank wall that is initially warm, and to accommodate liquid residuals in the tank after expulsion is stopped. With this modification, the existing correlation predicted mass requirements to within 14% of experimental results. Revision of the correlation constants using a nonlinear least-squares fit of the current experimental data has a minor effect, thus supporting the validity of the original correlation's form, its fitted constants, and the alternative definition of the tank equivalent diameter.

Nomenclature

A = surface area
C = ratio of wall-to-gas effective thermal capacity
CF = collapse factor, w_p/w_p^0
c_p = specific heat at constant pressure
D_{eq} = equivalent tank diameter
h_c = gas-to-tank wall free convection heat transfer coefficient
m = mass
$p_1 \ldots p_8$ = fitted constants
Q = ratio of total ambient heat input to effective thermal capacitance of gas
$\dot{q}$ = heat flux from ambient to tank wall
S = modified Stanton number
T_s = saturation temperature of propellant at initial tank pressure
T_0 = pressurant inlet temperature
t = thickness
w_p = total pressurant mass
w_p/w_p^0 = collapse factor
w_p^0 = total pressurant mass under conditions of zero heat and mass tranfer
V = volume
ΔV = expelled liquid volume
θ_T = total liquid outflow time
ρ = density
$\sim$ = computed value that accommodates variable wall thickness or material

Subscripts
exp = experimental
G = gas
lid = tank lid only
pred = predicted
sw = swept by the liquid free surface during expulsion
tank = tank wall excluding lid
w = wall

Superscript
0 = at pressurant inlet temperature and tank expulsion pressure

Introduction

THE pressurized expulsion of cryogenic fluids from propellant tanks was an active research area during the 1960s and early 1970s, as evident from the large number of publications on this subject. Discussed herein is the cryogenic pressurant requirements correlation developed by Epstein,[1] and subsequently revised by Epstein and Anderson.[2] The correlation predicts the collapse factor, a dimensionless pressurant mass, given the following dimensionless groups: pressurant-to-saturation temperature ratio, wall-to-gas effective thermal capacity ratio, ratio of ambient heat input to effective thermal capacitance of pressurant, and a modified Stanton number for gas-to-tank wall heat transfer. The original correlation was developed for cylindrical liquid hydrogen and oxygen tanks pressurized by the propellant vapor or helium. This form of the correlation has a theoretical basis and contains eight constants determined by nonlinear least-squares fitting of computed points from a pressurization computer program.[1,2] In the later paper,[2] the correlation was revised with updated constants to include axisymmetric tanks through the use of an equivalent tank diameter. The revised correlation was compared to experimental data from numerous sources and reported to agree to within ±12%, provided the data variables were within specified ranges.

The form of Epstein and Anderson's correlation is[2]

$$\frac{w_p}{w_p^0} = \left\{\left(\frac{T_0}{T_s} - 1\right)[1 - \exp(-p_1 C^{p_2})] \times [1 - \exp(-p_3 S^{p_4})] + 1\right\} \times \exp\left[-p_5\left(\frac{1}{1 + C}\right)^{p_6}\left(\frac{S}{1 + S}\right)^{p_7} Q^{p_8}\right] \tag{1}$$

where

$$w_p^0 = \rho_G^0 \Delta V \tag{2}$$

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$$C = \frac{(\rho c_p^0 t)_w}{(\rho c_p)_G^0 D_{eq}} \frac{T_s}{T_0} \tag{3}$$

$$S = \frac{h_c \theta_T}{(\rho c_p)_G^0 D_{eq}} \frac{T_s}{T_0} \tag{4}$$

$$Q = \frac{\dot{q} \theta_T}{(\rho c_p)_G^0 D_{eq} T_0} \tag{5}$$

The quantity w_p/w_p^0 is known as the collapse factor and represents the ratio of actual pressurant consumption to an ideal amount assuming no heat or mass transfer from the pressurant. The heat transfer coefficient in Eq. (4) is obtained from a Nusselt number correlation for turbulent free convection for vertical planes and cylinders.[3] The following is a list of Epstein and Anderson's values[2] of fitted constants for liquid hydrogen pressurized by either hydrogen or helium gas: p_1, 0.330; p_2, 0.281; p_3, 4.26; p_4, 0.857; p_5, 1.50; p_6, 0.312; p_7, 0.160; and p_8, 0.986.

Since publication of the revised correlation,[2] additional experimental data were obtained at the NASA Lewis Research Center for the pressurized expulsion of liquid hydrogen from spherical and nearly spherical tanks.[4–8] The data series and references are listed in Table 1.

A total of 60 data points are available from the sources in Table 1. These data points were obtained using a variety of pressurant gas diffusers. Data obtained with straight-pipe gas injectors were not included because this injector configuration results in high heat and mass transfer rates at the liquid surface. (For the present data set, the ratio of total mass transferred across the liquid–vapor interface-to-total pressurant mass ranged from −0.26 to 0.19, where a positive value represents condensation. Although this information is not generally known, one should be careful to apply the correlation in its present form only to conditions where mass transferred across the liquid–vapor interface is no greater than ±25% of the pressurant mass. Some cases where this condition is known not to hold are expulsion during liquid sloshing and expulsion of slush hydrogen.) With a few exceptions, the data variables fall within the ranges specified for the Epstein and Anderson correlation,[2] as shown in Table 2. The most significant differences are some data points having longer total outflow time and the low ambient heat flux for the NASA data. In this work, the Epstein and Anderson correlation[2] is compared to the NASA data and a revision of the correlation is provided.

Table 1 Liquid hydrogen expulsion data obtained at NASA

Data series	Reference	Tank diameter, m	Tank shape	Pressurant gas
I	Van Dresar and Stochl[4]	2.2	Oblate spheroid	GH_2
II	Stochl et al.[5]	4.0	Sphere	GH_2
III	Stochl et al.[6]	1.5	Sphere	GH_2
IV	Stochl et al.[7]	4.0	Sphere	GHe
V	Stochl et al.[8]	1.5	Sphere	GHe

Table 2 Range of variables for correlation

Variable	Epstein and Anderson[2]	NASA data
Spherical tank diameter, m	1.5–9.1	1.5–4.0
Wall thickness, cm	0.25–2.5	0.21–1.3
Pressurant inlet temperature-to-propellant saturation temperature ratio	2–15	8–17
Total outflow time, s	200–500	132–1980
Ambient heat flow, W/m^2	0–32,000	2.3–100

Comparison of Data to Epstein and Anderson Correlation

Although not stated, the Epstein and Anderson correlation[2] assumes that the tank is completely expelled, i.e., liquid residuals are zero. In the NASA experiments, expulsions were stopped at approximately 5% liquid fill level. Therefore, when comparing the predictions to the data, adjustments were made to correct for the liquid residuals. This correction was achieved by omitting the liquid residual volume and the mass of the corresponding tank wall from the analysis, i.e., the appropriate tank volume does not include the liquid residual volume and the appropriate tank mass does not include the mass of the tank wall that remained wetted at the conclusion of the experiments.

Epstein and Anderson[2] state that their correlation may be used when the initial ullage volume does not exceed 20% of the total tank volume. For the present data set, initial ullage volumes were from 5 to 14% of the tank volume after correcting for the liquid residual volume.

The correlation further assumes a uniform wall thickness and material. All of the NASA data were obtained in tanks fitted with lids that were thick compared to the tank walls, and in the case of data from Refs. 5–8, the lid material differed from that of the tank. The adjusted tank wall density, thickness, and specific heat capacity were obtained as follows:

$$\tilde{\rho}_w = \frac{m_{\text{tank}} + m_{\text{lid}}}{V_{\text{tank}} + V_{\text{lid}}} \tag{6}$$

$$\tilde{t}_w = \frac{m_{\text{tank}} + m_{\text{lid}}}{\tilde{\rho}_w A_w} \tag{7}$$

$$\tilde{c}_p = \frac{m_{\text{tank}} c_{p,\text{tank}} + m_{\text{lid}} c_{p,\text{lid}}}{m_{\text{tank}} + m_{\text{lid}}} \tag{8}$$

The adjusted values obtained from Eqs. (6–8) were then entered into Eq. (3) to calculate the C parameter.

Comparison of predicted and experimental results are shown in Fig. 1. The data points generally fall above the diagonal line representing perfect agreement. Specifically, the Epstein and Anderson correlation[2] predicts a greater collapse factor than the experimentally determined value for all but two points. Errors ranged from −4% to +27%, with a mean error of +15%. The rms error is 5.6%.

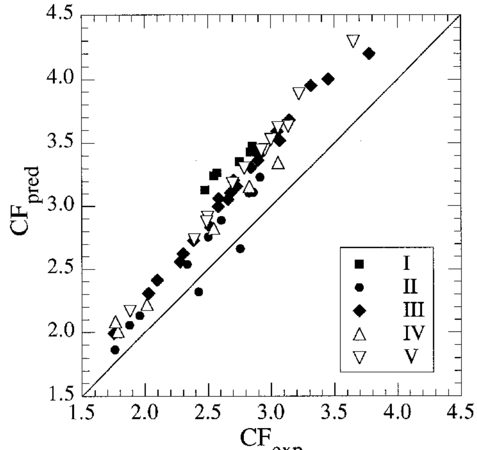

Fig. 1 Correlation results using Epstein and Anderson's definition[2] for equivalent diameter.

Table 3 Comparison of original and revised constants for the Epstein and Anderson[2] correlation for hydrogen propellant

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
Epstein and Anderson[2]	0.330	0.281	4.26	0.857	1.50	0.312	0.160	0.986
Revised constants	0.300	0.291	5.71	0.906	1.50	0.312	0.160	0.986

Modification of the Correlation

All of the NASA data were obtained with tank hardware having initially warm thermal mass concentrated at the top of the test tanks. It is suspected the major cause of the discrepancy in the previously discussed comparison is the inclusion of the warm thermal mass of the upper tank wall, tank neck, and lid. This mass is not initially at the cold saturation temperature of the propellant, but at elevated temperatures approaching that of the ambient temperature of the surroundings or of the pressurant gas inlet temperature. In test series II–V, the ullage was exposed to warm pressurant gas flow prior to the test during a gas temperature conditioning procedure. In test series I, there was no conditioning of the pressurant gas temperature; however, initial lid temperatures were near ambient temperature. Because this upper wall thermal mass is initially warm, it is not expected to absorb much thermal energy from the pressurant gas. Thus, it is reasonable to attempt to modify the correlation by excluding the initially warm thermal mass.

In their paper, Epstein and Anderson[2] defined the equivalent diameter as the "diameter of a cylindrical tank having the same wall surface area and total volume as the tank under investigation." Here, an alternative definition for the equivalent diameter is suggested

$$D_{eq} = 4\Delta V / A_{sw} \tag{9}$$

where A_{sw} is the area of wall surface swept by the liquid free surface during the expulsion process, i.e., the wall surface area initially wetted by the propellant that is exposed to gas at the end of the expulsion. For an initially full tank that is completely expelled, this definition is equivalent to Epstein and Anderson's definition.[2] Otherwise, this definition removes the influence of both liquid residuals and warm tank walls above the initial liquid level. When Epstein and Anderson's correlation[2] form and constants are used with the alternative equivalent diameter, much improved results are obtained (Fig. 2). Errors range from −10 to +14%, with a mean error of +0.6%. The rms error is 4.7%.

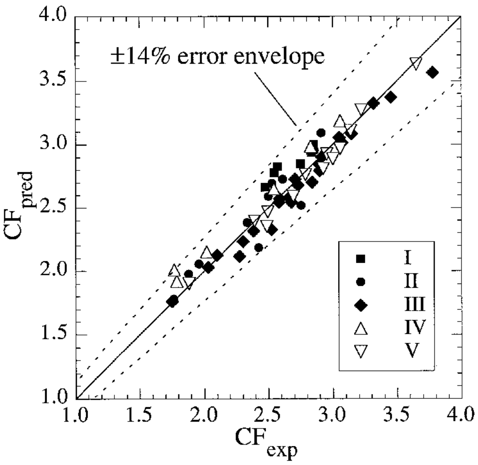

Fig. 2 Correlation with alternative definition for equivalent diameter.

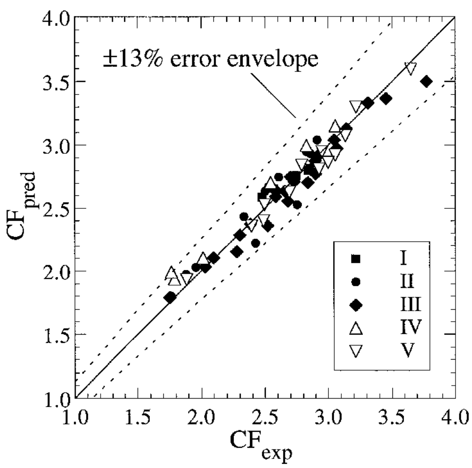

Fig. 3 Correlation with alternative definition for equivalent diameter and revised constants.

Revision of Fitted Constants

The constants p_1 through p_4 were updated using a nonlinear least-squares fit of the NASA data. Because the maximum ambient heat flow of the NASA data was less than one-half of 1% of the maximum from Epstein and Anderson's work,[2] no attempt was made to update their constants for the environmental heat input (constants p_5 through p_8). [The exponential multiplier in Eq. (1) containing the Q parameter has values ranging from 0.949 to 0.999 for the present data set. Therefore, its impact on the correlation is relatively small.] The comparison of predicted collapse factor with the experimental data is shown in Fig. 3, and the revised constants are listed in Table 3 along with Epstein and Anderson's values.[2] The revised constants give a slightly smaller error envelope and rms error. Errors range from −8 to +13%, with a mean error of +0.5%. The rms error is 4.1%. Note that the revised constants reduce the error envelope, but only by an incremental amount. This indicates that the use of the alternative definition of equivalent diameter with the original Epstein and Anderson correlation[2] has merit and can be used with confidence.

Conclusions

The correlation of Epstein and Anderson[2] is considered reliable for axisymmetric liquid hydrogen tanks, provided one remains within the specified range of variables, the initial ullage space is not more than 20%, the liquid is completely expelled, and the tank wall initially above the liquid level is near the saturation temperature.

If the liquid is not completely expelled, or if the upper tank wall temperatures are significantly above the propellant saturation temperature, then the alternative definition of equivalent diameter presented within should be employed. The portion of thermal mass initially at elevated temperatures with respect to the saturation temperature or thermal mass below the final liquid level should be excluded when calculating the C parameter [Eq. (3)] and equivalent diameter [Eq. (9)].

The correlations do not contain dimensionless groups that quantify heat and mass exchange between the pressurant and the propellant. Therefore, the correlations should not be used to predict pressurant mass requirements in systems where these effects are relatively large, e.g., systems with liquid sloshing or slush hydrogen systems.

The ±13 to ±14% error envelope of the present work compares favorably with the ±12% error envelope reported by Epstein and Anderson.[2] The correlations are useful tools for estimating pressurant mass requirements in axisymmetric liquid hydrogen tanks.

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